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Kinematics for the Scattering of Spacelike Momentum

H. LEMKE

Ul. Obrutcheva, 4-1-51, *Moscow* 117 421, *U.S.S.R.*

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Abstract

The kinematics of elastic and inelastic tachyon–bradyon and tachyon–tachyon scattering $2 \to 2$ is examined. To get kinematic limits finite, the initial and final momenta of a tachyon are subjected to the restriction $pp' < 0$. This is sufficient in tachyon–bradyon scattering; in tachyon–tachyon scattering additional covariant restrictions exist, though these follow from the problem. In an alternative approach, where there is interaction the particle energies must be positive relative to the rest frame of the total momentum, which additional condition is shown to lead to a simplification of the kinematic results.

1. *Introduction*

It has been postulated that faster-than-light particles (tachyons) from conventional subluminal sources move in Minkowski space for all these sources (Lemke, 1975). According to this the kinematics for the scattering $2 \to 2$ of such tachyons from ordinary particles or other such tachyons must be Lorentz covariant. It has also been argued that the norm sign of any particle momentum does not change in an elastic collision (Lemke, 1977). We will therefore examine the two cases in which the particles do not change the 4-vector type.

In both reactions the initial total momentum can also be spacelike and lightlike. It is well known (Byckling and Kajantie, 1973) that the phase space for the final states is then infinite, which is unphysical. Therefore, in addition to 4-momentum conservation, covariant constraints must exist to bound the phase space. Clearly, such constraints can easily be postulated, but this is not our aim; they must be as few as possible and have a natural basis.

Recently, in treating electromagnetic tachyon-bradyon scattering, the restriction $pp' < 0$ was found to be the only natural restriction for the scalar product of the initial and final tachyon momentum and sufficient to limit the final momenta (Lemke, 1976). In Section 2 the kinematic limits will be given in the Mandelstam plane *s*–*t*–*u*.

This restriction could also be applied successfully in the quantum electrodynamics of superluminal currents (Lemke, 1976), and Section 3 will show that it is also useful in tachyon–tachyon scattering. In this reaction additional restrictions are necessary. But these will follow from the working hypothesis "that which is unphysical cannot be realized by nature." According to this philosophy negative-mass bradyons, for example, cannot be detected by future-oriented observers because there are apparently no means for limiting the phase space of the final states at $s = (p_1 + p_2)^2 < 0 + 0$. Without making further assumptions, we shall find that two tachyons can only interact if s is positive; at $s = 0$ the physical phase space is the limiting case of $s \to 0 + 0$.

In Subsection 3.4 we postulate that the signs of the particle energies are equal relative to the rest frame of the total momentum (the signs are positive by definition of the rest frame) and describe the consequences. This postulate can be regarded as related to a generalized concept of causality that also allows for a causal tachyon behavior (Lemke, 1977). That topic is beyond the scope of this paper; let me only mention that the postulate leads to the satisfactory result that proton and electron are stable under tachyonic decays.

For inelastic tachyon–tachyon scattering we shall find two selection rules, which, however, will not exist if the alternative approach in Subsection 3.4 applies.

There will be no upper limit for the final particle masses in inelastic scattering, whereas in bradyon–bradyon scattering the bound $(m_1' + m_2')^2 \leqslant s$ exists. This result is not unphysical because dynamical laws could rule out the excitation of in-coming particles to infinite rest masses. Nevertheless, we will show that the results in Subsection 3.4 afford such upper limits for tachyon–bradyon scattering and that these results can be used to find the upper limits for tachyon–tachyon scattering.

2. *Tachyon*$_1$ + *Bradyon*$_2$ $\to$ *Tachyon*$_1'$ + *Bradyon*$_2'$

2.1. *Elastic Scattering.* In relation to the initial bradyon's rest system the variable s can be written as

$$s = (p_1 + p_2)^2 = M^2 - m^2 + 2Me_1 \tag{2.1}$$

where M is the bradyon mass and m is the tachyon mass. Because of Lorentz covariance the tachyon energy e_1 cannot be subjected to bounds, and s can vary between $\pm$ infinity. So there is no center-of-mass system (c.m.s.) among the Lorentz frames for some values of e_1, and the rest system of the initial bradyon proves to be the only natural Lorentz frame. In the following all momentum variables will refer to this system.

The squared invariant momentum transfer reads

$$t = (p_2 - p_2')^2 = -2M(e_2' - M) \tag{2.2}$$

It is always negative since $e_2' \geqslant M$. Squaring the conservation law $p_1' = p_1 + p_2 - p_2'$ one finds $\mathbf{p}_1\mathbf{p}_2' = e_2'(M + e_1) - M(M + e_1)$, and from squaring once more, the dependence of the final bradyon energy e_2' on e_1 and $\cos\theta_2$ follows:

$$e_2' = M\frac{(e_1 + M)^2 \pm (e_1{}^2 + m^2)\cos^2\theta_2}{(e_1 + M)^2 - (e_1{}^2 + m^2)\cos^2\theta_2} \tag{2.3}$$

Because the lower sign gives $e_2' = M$ (no interaction), the upper sign is valid. Energy e_2' increases with $\cos^2\theta_2$ and reaches its maximum at $\cos^2\theta_2 = 1$ if $e_1^2 + m^2 < (e_1 + M)^2$, which condition equates with $s > 0$ [see (2.1)]. If $s < 0$, there is an upper boundary of $\cos^2\theta_2$, where e_2' tends to infinity. Hence variable t has a lower limit only where $s > 0$. By substituting $e'_{2\,\max}$ into (2.2) one finds

$$t^-(s) = -s^{-1}[s^2 + 2s(m^2 - M^2) + (m^2 + M^2)^2] \tag{2.4}$$

This is a hyperbolic branch that reaches to $-\inf$ at the limits $s \to 0$ and $s \to \inf$, the maximum being located at $s_{1,2} = \pm(m^2 + M^2)$. At s_1 the value of (2.4) reads

$$t^-(s_1) = -4m^2 \tag{2.5}$$

For the sum of the Mandelstam variables we have $s + t + u = 2(M^2 - m^2)$, which is positive or negative for $M > m$ or $M < m$, respectively. Substituting the kinematic limit (2.4) one gets this limit in the form $us = (m^2 + M^2)^2$, which, as also the above relations, is the same as in bradyon-bradyon scattering after changing the sign of m^2.

The lower limit for t at negative s is provided by the restriction $p_1p_1' < 0$, which leads to

$$t > -2m^2 \tag{2.6}$$

This also is seen to cut off the singularity of (2.4) at vanishing s. Because of (2.5) kinematic limit (2.6) lies above the bound (2.4) that has followed from only the conservation of 4-momentum and the physical condition $e_2' \geqslant M$.

The results are summarized in the Mandelstam plane in Figure 1. The essential difference between the depicted kinematic region and that in bradyon-bradyon scattering is its independence of s. Therefore, as (2.2) shows, the phase space of the final bradyon energy is also s independent; it is given by

$$1 + m^2/M^2 > e_2'/M > 1$$

according to which e_2'/M can vary the more the greater the tachyon mass in units of the bradyon mass. Moreover, the kinematic regions of the s and u channels are the same. Experience shows that a t channel does not exist. Besides, one sees from Figure 1 that restriction (2.6) would not be necessary for bounding the phase space if one was able to require that $s \geqslant M^2 + m^2$.

2.2. *Inelastic Scattering.* Instead of (2.2) we now have $t = M^2 + M'^2 - 2Me_2'$, which is smaller than $(M' - M)^2$ because $e_2' \geqslant M'$. To find the upper and lower limits $t \pm (s)$, we consider them in the c.m.s. There

$$t = M^2 + M'^2 - 2(e_2e_2' - \mathbf{p}_2\mathbf{p}_2') \tag{2.7}$$

$$e_2 = \tfrac{1}{2}s^{-1/2}(s + m^2 + M^2), \qquad e_1 = \tfrac{1}{2}s^{-1/2}(s - m^2 - M^2) \tag{2.8}$$

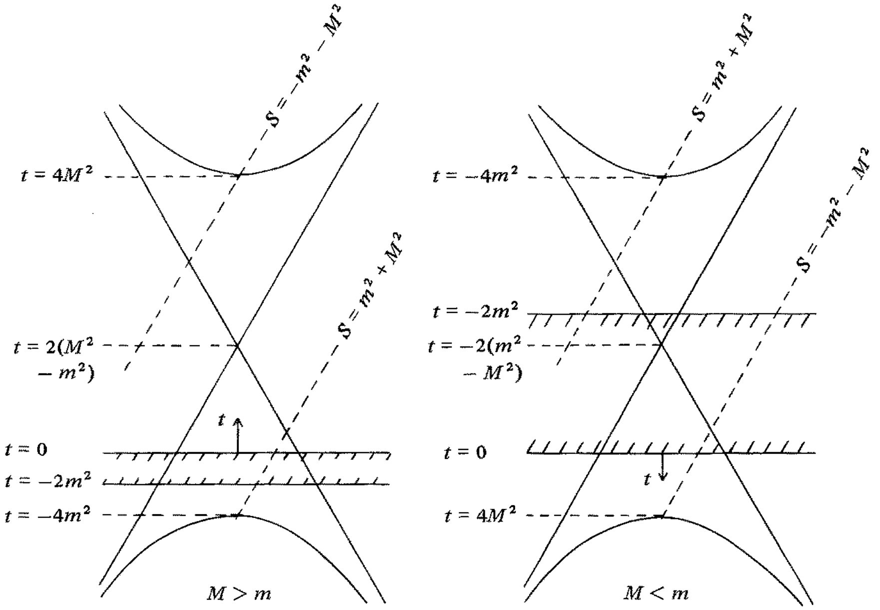

Figure 1. Physical regions in elastic tachyon–bradyon scattering, where m is the tachyon mass.

Expression (2.7) has its maximum and minimum for parallel and antiparallel $\mathbf{p}_2$ and $\mathbf{p}_2'$, since these momenta have fixed magnitudes. Hence,

$$t\pm = M^2 + M'^2 - \tfrac{1}{2}s^{-1}\{(s + m^2 + M^2)(s + m'^2 + M'^2) \mp [(s + m^2 + M^2)^2 - 4M^2 s]^{1/2}[(s + m'^2 + M'^2)^2 - 4M'^2 s]^{1/2}\} \quad (2.9)$$

One can convince oneself that t^- equals (2.4) and $t^+ = 0$ for elastic scattering.

The upper bound $t^+(s)$ is found to tend to zero for $s \to \pm\text{inf}$, i.e., the change of rest mass does not play any part in this limit. At $s = 0$ one finds the finite value

$$t^+(s = 0) = (M'^2 m^2 - M^2 m'^2)\left(\frac{1}{m^2 + M^2} - \frac{1}{m'^2 + M'^2}\right) \quad (2.10)$$

So t^+ varies within certain finite boundaries. It is defined at every s because the roots in (2.9) are seen to remain real.

Let us examine the case of equal masses, $m = M$ and $m' = M'$, more closely. The Taylor series of t^+ about $s^{-1} = 0$ begins with $t^+(s \to \pm\text{inf}) = (m'^2 - m^2)^2/s$, which shows that t^+ approaches zero from above for $s \to +\text{inf}$ and it approaches zero from below for $s \to -\text{inf}$. The expansion about $s = 0$ is of the form

$$t^+(s \to 0) = \tfrac{1}{4}s\left(\frac{m'^2}{m^2} + \frac{m^2}{m'^2} - 2\right)$$

Because the parameter is positive-definite, t^+ increases with s in the vicinity

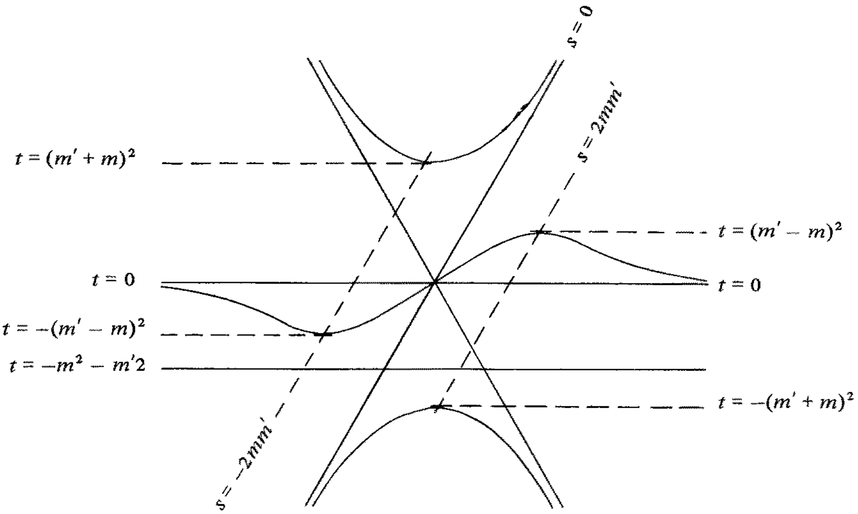

Figure 2. Inelastic tachyon–bradyon scattering where $M = m$ and $M' = m'$.

of $s = 0$. Hence at least one minimum and one maximum exist. Differentiating with respect to s one finds their positions at $s_{1,2} = \pm 2mm'$. So there are one minimum and one maximum, whose values can be shown to be $t_1^+ = (m' - m)^2$ and $t_2^+ = -(m' - m)^2$. (See Figure 2.)

The lower bound $t^-(s)$ again jumps across $\pm$inf at $s = 0$. In our special case of equal initial and equal final masses the local maximum and minimum of t^- are found at $s_{1,2} = \pm 2mm'$, at which positions t^- takes on the values $t_1^- = -(m' + m)^2$ and $t_2^- = (m' + m)^2$. This shows that t^- lies again below the kinematic limit $t > -m^2 - m'^2$ that follows from $p_1 p_1' < 0$.

3. *Tachyon*$_1$ + *Tachyon*$_2$ $\rightarrow$ *Tachyon*$_1'$ + *Tachyon*$_2'$

3.1. As in tachyon–bradyon scattering the total momentum is timelike ($s > 0$), lightlike ($s = 0$), or spacelike ($s < 0$). Only if it is timelike, there is one Lorentz frame for which the initial (and final) three-momenta are antiparallel and of equal magnitude. If the initial three-momenta are only antiparallel, then s is not necessarily positive, whereas when they are parallel, s is negative.

Let us give an elementary proof that there is not necessarily a Lorentz frame for which the initial three-momenta are colinear. Assuming such a Lorentz frame exists, the frames moving parallel to the direction of the momenta would also be such frames, among them the standard system of one of the momenta, say p_1 [$p_{1\text{stan.}} = (0, m, 0, 0)$], which system is determined up to rotations and Lorentz boosts in the plane perpendicular to $\mathbf{p}_1$ [the elements of $O(1, 2)$]. Therefore, if such a Lorentz frame existed, there would also have to be one frame among the p_1 standard systems for which $\mathbf{p}_1$ and $\mathbf{p}_2$ are colinear. This, however, is not necessarily the case. To see this, we denote the projection of $\mathbf{p}_2$ on the plane perpendicular to $\mathbf{p}_1$ by $\mathbf{p}_{2t}$. The Lorentz boost along directions in this plane can map this projection onto zero only if $|\mathbf{p}_{2t}|/e_2 < 1$, which is not necessarily satisfied. In particular, this is not satisfied if $\mathbf{p}_2$ is perpendicular to $\mathbf{p}_1$ in the p_1 standard system.

3.2. *Elastic Scattering.* The sum of the Mandelstam variables takes on the value $s + t + u = -2m_1^2 - 2m_2^2$, which is always negative.

Let us begin with negative s. In the standard frame of the total momentum the law of four-momentum conservation reads

$$\begin{aligned} 0 &= \mathbf{E}_1 + \mathbf{E}_2 = \mathbf{E}_1' + \mathbf{E}_2' \\ \pm(-s)^{1/2} &= p_{1x} + p_{2x} = p_{1x}' + p_{2x}' \end{aligned} \tag{3.1}$$

where the three-vectors in the (0, 2, 3)-hyperspace have been designated by $\mathbf{E}$, $E^2 = p_x^2 - m^2$. By squaring $\pm(-s)^{1/2} - p_{1x} = p_{2x}$ one finds $p_{1x}, \ldots, E_1, \ldots$ as functions of s

$$\begin{aligned} p_{1x} = p_{1x}' &= \pm\tfrac{1}{2}(-s)^{-1/2}(m_1^2 - m_2^2 - s), \\ p_{2x} = p_{2x}' &= \pm\tfrac{1}{2}(-s)^{-1/2}(m_2^2 - m_1^2 - s) \end{aligned} \tag{3.2}$$

Equations (3.1)-(3.2) show that $E_1^2 = E_2^2 = E_1'^2 = E_2'^2$. According to (3.2) $|p_{ix}| > m_i$ for $0 > s > -(m_1 - m_2)^2$ and $s < -(m_1 + m_2)^2$; between these intervals $|p_{ix}| < m_i$ is valid. Correspondingly, $E_i^2 > 0$ in the first case and $E_i^2 < 0$ in the second.

At first we will deal with the second case and consider

$$t = -2(m_1^2 + \mathbf{E}_1\mathbf{E}_1' - p_{1x}p_{1x}') \tag{3.3}$$

Because of $E_1^2 = E_1'^2 < 0$ the vectors $\mathbf{E}_1$ and $\mathbf{E}_1'$ lie in the (0, 2, 3)-hyperspace on the same spacelike hyperboloid, which is single sheeted. Because the product $\mathbf{E}_1\mathbf{E}_1'$ is invariant under the elements of $O(1, 2)$, there is a frame S^+ for which $\mathbf{E}_1 = (0, p_{1y}^+, 0)$ and

$$\mathbf{E}_1\mathbf{E}_1' = -(-E_1^2)^{1/2}p_{1y}'^+$$

$p_{1y}'^+$ can take on every value between $\pm$inf, because of which t is unbounded. Moreover, the planes $p_{1y}'^+ =$ const intersect the spacelike hyperboloid on a pair of parabolas on which $e_1'^+$ and $p_{1z}'^+$ vary without limit. Hence the phase space of the secondaries is infinite even for given s and t. In other words, no restriction on the invariants p_1p_2' or p_1p_1' can serve for bounding the magnitudes of the final momenta. We conclude that there is no interaction if $-(m_1 - m_2)^2 > s > -(m_1 + m_2)^2$.

We turn to the region $0 > s > -(m_1 - m_2)^2$, $-(m_1 + m_2)^2 > s$. Here $E_2^2 = E_2'^2$ is positive, so $\mathbf{E}_2$ and $\mathbf{E}_2'$ lie in the (0, 2, 3)-hyperspace on the same timelike hyperboloid, which is two sheeted. As $\mathbf{E}_2'$ can equal $\mathbf{E}_2$ (then $t = 0$, no interaction) both vectors lie on the same sheet. By this $\mathbf{E}_2\mathbf{E}_2' > E_2^2$, so (3.3) is negative-definite. The restrictions $p_ip_i' < 0$ provide the lower boundary

$$t > -2\min(m_1^2, m_2^2) \tag{3.4}$$

Thus, we have arrived at a finite physical t-region. But as shown above, this is not sufficient for having a finite phase space. Indeed, in the rest frame S^+ of $\mathbf{E}_2$

$$\mathbf{E}_2\mathbf{E}_2' = E_2 e_2'^+$$

and in the limit $s \to -(m_1 + m_2)^2 - 0$ or $s \to -(m_1 - m_2)^2 + 0$, in which E_2 vanishes, $e_2'^+$ has to tend towards infinity to decrease (3.3) to the lower boundary (3.4). That is, in this limit the boundaries for the magnitudes of the final momenta diverge to infinity.

As there are no means to exclude $s = -(m_1 \pm m_2)^2$ from the physical region, interaction cannot happen if s is negative. In other words, if in the standard frame of p_1 or p_2 the initial three-momenta obey $\mathbf{p}_1\mathbf{p}_2 > -\frac{1}{2}(m_1{}^2 + m_2{}^2)$, interaction will be impossible. Analogously, bradyons traveling backward in time (negative mass) would not interact with those traveling forward in time (positive mass) (Section 1). Such a necessary absence of interaction (which is a phenomenon unknown in bradyon physics) already follows from general considerations of causality (Lemke, 1977).

We now come to a positive s. Here a c.m.s. exists, relative which

$$\begin{aligned} \pm s^{1/2} &= e_1 + e_2 = e_1' + e_2' \\ 0 &= \mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}_1' + \mathbf{p}_2' \end{aligned} \tag{3.5}$$

where $\mathbf{p}^2 = e^2 + m^2$. These equations yield the dependences

$$e_1 = e_1' = \pm\tfrac{1}{2}s^{-1/2}(s + m_2{}^2 - m_1{}^2), \qquad e_2 = e_2' = \pm\tfrac{1}{2}s^{-1/2}(s - m_2{}^2 + m_1{}^2) \tag{3.6}$$

and the equations $p_1 = p_1' = p_2 = p_2'$. Instead of (3.3) we can write

$$t = -2(m_1{}^2 + e_1 e_1' - \mathbf{p}_1\mathbf{p}_1') \tag{3.7}$$

which has its maximum for parallel $\mathbf{p}_1'$ and $\mathbf{p}_1$ (where $t = 0$) and its minimum for antiparallel $\mathbf{p}_1'$ and $\mathbf{p}_1$. This minimum is a negative-definite function $t(s)$ that follows from substituting (3.6); it is given by

$$u = (m_1{}^2 - m_2{}^2)^2/s \tag{3.8}$$

like in bradyon–bradyon scattering, since only the signs of the squared rest masses must be changed. (See Figure 3.) At vanishing s this boundary shows an unphysical divergence which is just cut off by the requirement $p_i p_i < 0$, leading to (3.4). The inequalities $0 > t >$ (3.4) and (3.7) give the boundaries for p_{ix}', the projection of $\mathbf{p}_i'$ on $\mathbf{p}_i$,

$$0 < 1 - p_{ix}'/|\mathbf{p}_i| < m_1{}^2/|\mathbf{p}_i|^2 \tag{3.9}$$

in the c.m.s., where $m_1 < m_2$ has been chosen.

By (3.6) the phase space of the final states is bounded at least outside the singularity at vanishing s. Let us inquire into this limit. In the case $m_1 = m_2$, in which $e_1 = e_2 = e_1' = e_2'$ tend towards zero and all three-momenta are of magnitude m and satisfy $\mathbf{p}_i\mathbf{p}_i' > 0$, the phase space of a final momentum is seen to consist of half the sphere with radius m in three-momentum space relating to c.m.s. This agreeable result does not generally follow at $s = 0$. At

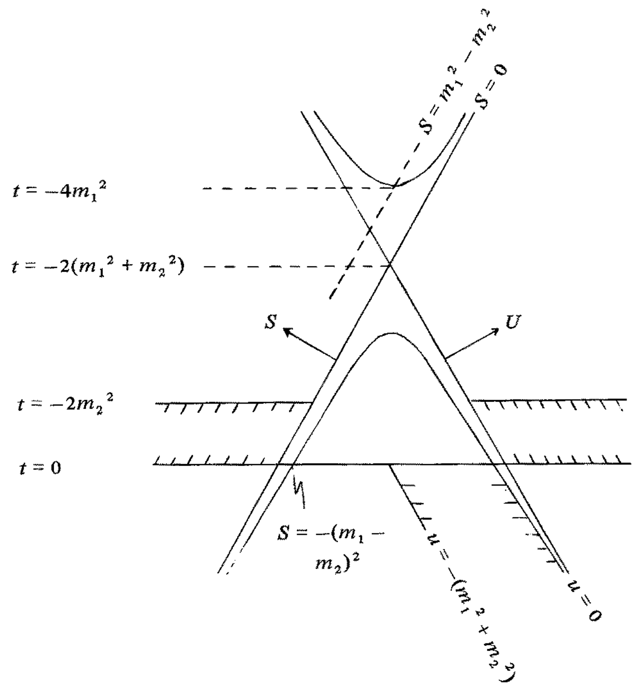

Figure 3. Physical regions in tachyon–tachyon scattering, $m_2 < m_1$. Note the crossing symmetry where $m_1 = m_2$.

this value $p_1' = -p_2'$ is also possible without any limit on the magnitudes of the components. Therefore, the physical phase space of the secondaries at $s = 0$ is obtained as the limiting case of

$$s \to 0 + 0 \tag{3.10}$$

Let us see what happens at this limit if $m_1 \neq m_2$. For $s \ll |m_1^2 - m_2^2|$ (3.6) gives $e_1 = e_1' = -e_2 = -e_2'$, which energies increase in magnitude like $s^{-1/2}$ in the c.m.s. But by supposition, the components of p_1 and p_2 have normal magnitudes for all the Lorentz frames that do not move in the vicinity of the light cone. So the more the magnitudes increase the more the c.m.s. moves in the vicinity of the light cone relative to these Lorentz frames; that is, the c.m.s. becomes an unphysical frame for vanishing s where $m_1 \neq m_2$. [As (3.9) shows, in this frame p_{ix}' approaches $|\mathbf{p}_i| = |\mathbf{p}_i'|$. In other words, the final and initial states become identical and the phase space of the final momenta vanishes at limit (3.10) in the c.m.s.]

However, the standard system of p_2 is physical for every value of s (provided that e_2 is not very high relative to Lorentz frames that do not move in the vicinity of the light cone). The transition to the p_2 standard system is given by the Lorentz boost

$$e^s = \frac{p_2}{m_2}\left(e - \frac{e_2}{p_2} p_x\right) \tag{3.11}$$

$$p_x{}^s = \frac{p_2}{m_2}\left(p_x - \frac{e_2}{p_2}\,e\right) \tag{3.12}$$

(These equations can be checked with $p_x = p_2$ and $e = e_2$.) Since $e_2 = e_2'$ in the c.m.s., the transformation (3.11) of p_2' can be rewritten as $e_2'^s = p_2 e_2(1 - p_{2x}'/p_2)/m_2$, which together with (3.9) ($m_2 > m_1$) shows that $|e_2'^s|$ varies within the limits

$$0 < |e_2'^s| < m_1{}^2|e_2|/m_2 p_2 \tag{3.13}$$

and if $m_1 > m_2$

$$0 < |e_2'^s| < m_2|e_2|/p_2 \tag{3.14}$$

Because the right-hand side is finite for vanishing s (and equal to either $m_1{}^2/m_2$ or m_2) the statement before (3.10) has general validity.

3.3. Let us calculate the kinematic limit of energy e_2' relative to the standard frame of p_2 directly. Squaring the conservation law $p_1' = p_1 + p_2 - p_2'$ we find $e_1 e_2' = \mathbf{p}_2'(\mathbf{p}_1 + \mathbf{p}_2) - \mathbf{p}_2(\mathbf{p}_1 + \mathbf{p}_2)$. With the angular variable

$$z' = \frac{1}{e_1 p_2'}\,\mathbf{p}_2'(\mathbf{p}_1 + \mathbf{p}_2) \tag{3.15}$$

and the given parameter

$$z = \frac{1}{e_1 m_2}\,\mathbf{p}_2(\mathbf{p}_1 + \mathbf{p}_2) = \frac{1}{e_1 m_2}\,(\mathbf{p}_2\mathbf{p}_1 + m_2{}^2) \tag{3.16}$$

this relation becomes

$$e_2' = p_2' z' - m_2 z \tag{3.17}$$

Squaring once more yields for e_2' the two solutions

$$e_2' = \frac{m_2}{z'^2 - 1}\,[z \pm z'(1 + z^2 - z'^2)^{1/2}] \tag{3.18}$$

Let us consider the p_2 standard frame with respect to which $\mathbf{p}_1$ and $\mathbf{p}_2$ are antiparallel and discuss the behavior of z. (Such a Lorentz frame exists for every positive s, following from boosting along $\mathbf{p}_2$ in the c.m.s.) For this standard frame $s > 0$ equates with

$$p_1 > (m_1{}^2 + m_2{}^2)/2m_2 \tag{3.19}$$

Furthermore, (3.16) takes on the form

$$z(p_1) = \pm(m_2 - p_1)/(p_1{}^2 - m_1{}^2)^{1/2} \tag{3.16'}$$

in this standard frame. This function tends to ± 1 for p_1 tending to infinity, it becomes infinite for $p_1 \to m_1$, and it is found to equal

$$z[(3.19)] = \begin{cases} \pm 1 & \text{for } m_1 \neq m_2 \\ 0 & \text{for } m_1 = m_2 \end{cases} \tag{3.20}$$

at the lower limit (3.19), where $s = 0$. Because of these boundaries

$$|z| \leqslant 1 \tag{3.21}$$

in the physical p_1 region. Function $d\,z(p_1)/dp_1$ is found to be zero at $p_1 = m_1{}^2/m_2$, which value is withing the physical p_1 region when $m_2 < m_1$. In this case $|z|$ has a local minimum. In case $m_2 > m_1$, function z has a zero at $p_1 = m_2$, and strictly monotonously varies between +1 and −1.

According to the limits of variable t the restrictions

$$0 < p'_{2x} < m_2 \qquad \text{or} \qquad (m_2{}^2 - m_1{}^2)/m_2 < p'_{2x} < m_2 \tag{3.22}$$

are valid in the cases $m_2 \leqslant m_1$ and $m_2 \geqslant m_1$, respectively. The upper boundaries are characterized by $t = 0$. As we see from (3.7), which is related to the c.m.s., $p'_{2x} = p'_2$ only at this value, and as we see from (3.15), (3.16), and (3.16′) only then does $z' = z$. At $p'_{2x} = p_2$ we have $e'_2 = 0$. Hence, the physical solution in (3.18) must equal zero at $z' = z$; only the second solution obeys this (minus sign).

The sum $\mathbf{p}_1 + \mathbf{p}_2$ in (3.15) has the components $(m_2 - p_1, 0, 0)$. When $m_2 \leqslant m_1$ component $m_2 - p_1$ is negative and therefore the left relation (3.22) leads us to

$$e_1 z < e_1 z' < 0 \tag{3.23}$$

Where $m_2 \geqslant m_1$ component $m_2 - p_1$ can be positive and negative. If it is negative,

$$e_1 z < e_1 z' < e_1 z(m_2{}^2 - m_1{}^2)/m_2 p'_2 \tag{3.24}$$

follows. If positive,

$$e_1 z(m_2{}^2 - m_1{}^2)/m_2 p'_2 < e_1 z' < e_1 z \tag{3.25}$$

which inequalities are written so that they can be used for positive and negative e_1. z' clearly has the same sign as z and is smaller in magnitude than $|z|$, which is less than unity [see (3.21)].

The physical solution in (3.18) (sign minus) has no singularity at $z' = +1$ for $z > 0$ and $z' = -1$ for $z < 0$: one finds that $e'_2 = \frac{1}{2} m_2(z^{-1} - z)$ at $z' \to 1$, $z > 0$ and $z' \to -1$, $z < 0$. At the other singularity ($z' = +1$ or -1) e'_2 jumps across ±inf. The variable e'_2 leaves the real region at the limits $z'^2 = 1 + z^2$ and has the value $e'_2 = m_2/z$ there; $e'_2 = 0$ at $z = z'$ and $e'_2(z' = 0) = -m_2 z$. The function $e'_2(z')$ is sketched for a negative z in Figure 4.

To find the meaning of the right and left bounds in (3.24)-(3.25), respectively, we must compare them with (3.17), which leads to the upper ($z < 0$) or lower ($z > 0$) limit $e'_2 \lesseqgtr -z m_1{}^2/m_2$, where $m_2 \geqslant m_1$ [cf. (3.13)]. As expected, this limit is smaller than $-m_2 z$ in Figure 4. If $m_2 - p_1 < 0$ bounds (3.23)-(3.24) with $e_1 > 0$ are valid (for $z < 0$) in Figure 4, so z' is allowed to vary only between z and 0. There e'_2 is finite for every physical z (3.21). If $m_2 - p_1$ is positive, (3.25) with $e_1 < 0$ is valid, which marks the same interval as (3.24). For a positive z the results are analogous.

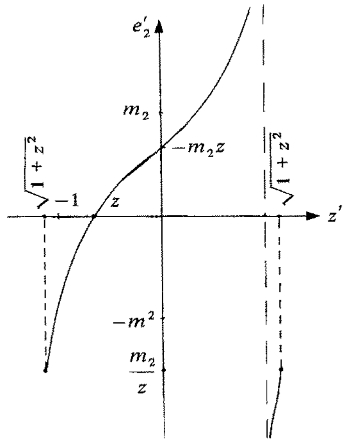

Figure 4. The final tachyon energy e_2' as a function of z' in the p_2 standard system relative to which $\mathbf{p}_1$ and $\mathbf{p}_2$ are antiparallel. The singularity at $z' = 1$ lies outside the kinematic region $z \leqslant z' \leqslant 0$.

3.4. *An Alternative Approach.* To see the s dependence of the e_2' limit depicted in Figure 4, we must look back to (3.16′)-(3.21). Where $m_1 = m_2$, $-z$ decreases with s and vanishes at $s = 0$, only $e_2' = 0$ remains physical.

Where $m_2 < m_1$, $-z$ equals 1 at $s = \inf$, passes a positive minimum and approaches 1 again as s vanishes. In case $m_2 > m_1$, $-z$ can have the maximum 1 at $s = 0$. It passes zero at an intermediate value of s. Then Figure 4 must be inverted with respect to the e_2'- and z'-axis, parameter $-z$ further decreases for $s \to \inf$ and approaches -1.

Clearly the kinematics would be simpler in the two cases of unequal masses if there were a lower boundary for s. For reasons of symmetry this boundary should be located where $-z$ is minimal in the first case and where $z = 0$ in the second. Indeed, such a lower bound results from the rule designed earlier for tachyonic particle decays (Lemke, 1977): Relative to the rest system of the initial (final) total momentum the final (initial) particle energies are required to be positive. Taking (3.6) into consideration, this leads to

$$|m_1^2 - m_2^2| < s$$

[For the lower signs in (3.6) the transition between the c.m.s. and this rest system S^+ is preformed by $e = -e^+$.] At the lower limit the initial energy of the one particle equals zero but the energy magnitude of the other has its minimum. Therefore, this limit gives just the restriction required for the simpler kinematics above.

In applying the rule to tachyon-bradyon scattering, two cases have to be distinguished: positive s (the rest system is a Lorentz frame) and negative s (the rest system is superluminally moving). In the first case (2.8) is valid and the rule leads to

$$\max\,(m'^2 + M'^2, m^2 + M^2) < s$$

(cf. Figure 1). In the second case we must know the equations of the transformation between the superluminal rest system S^+ and a Lorentz frame, e.g., the standard frame of the total momentum, relative to which expressions analogous to (3.2) hold good. For this Lorentz frame the transformation equations are given by $p_x = \pm e^+$ (Lemke, 1975, 1976; Parker, 1969), which shows that the requirement $e_i^+ > 0$ gives the boundary

$$s < -\max(m'^2 + M'^2, m^2 + M^2)$$

These boundaries show that m' and M' are bounded by $M'^2 + m'^2 < s$.

For various reasons, this alternative approach should refer only to particle decays. It is inoperative in elastic scattering but can apply in inelastic scattering.

3.5. *Inelastic Scattering.* Instead of (3.4) and (2.9) we have

$$t > -\min(m_1^2 + m_1'^2, m_2^2 + m_2'^2) \tag{3.4'}$$

$$\begin{aligned} t\pm = -m_1^2 - m_1'^2 - \tfrac{1}{2}s^{-1}\{&(s + m_2^2 - m_1^2)(s + m_2'^2 - m_1'^2) \\ &\mp [(s + m_2^2 - m_1^2)^2 + 4sm_1^2]^{1/2}[(s + m_2'^2 - m_1'^2)^2 + 4sm_1'^2]^{1/2}\} \end{aligned} \tag{3.26}$$

These functions are defined only outside the s interval

$$\max[-(m_1 - m_2)^2, -(m_1' - m_2')^2] > s > \min[-(m_1 + m_2)^2, -(m_1' + m_2')^2]$$

This shows that $t\pm$ can intersect the line $s = 0$ only if $m_1 \neq m_2$ and $m_1' \neq m_2'$. If this is satisfied, we find that $t^+(s \to 0) \to \inf$ for $(m_2 - m_1)(m_2' - m_1') < 0$ (asymmetric mass change). For $(m_2 - m_1)(m_2' - m_1') > 0$ (symmetrical mass change), we find $t^+(s = 0)$ to be given by (2.10) with M^2 and M'^2 substituted by $-m_1^2$ and $-m_1'^2$.

The first case leads to an infinite phase space at $s = 0$, so asymmetric mass change is kinematically interdicted. In the second case the less the difference between $m_2'^2$ and $m_1'^2$ the greater $t^+(s = 0)$, but it is finite because of the assumption $m_2'^2 \neq m_1'^2$. Because of (3.4′) and because t^+ approaches zero where s tends to infinity, the t region is finite where the mass change is symmetrical. Hence the phase space, too, is finite since s is positive.

If the initial masses are equal and the final masses are equal, (3.26) gives $t^+(s = 0) = -(m_1' - m_1)^2$. This case is depicted in the t channel in Figure 3.

Now let us examine the case $m_1 = m_2$, $m_1' \neq m_2'$. We find that

$$t^+(s \to 0) = -m_1^2 - m_1'^2 - \tfrac{1}{2}(m_2'^2 - m_1'^2) + s^{-1/2}m_1|m_2'^2 - m_1'^2| \tag{3.27}$$

This tends to +infinity, and hence there is no limited t region at $s = 0$. However, this case differs qualitatively from the two previous cases at $s = 0$. In the first, the initial total momentum is and becomes isotropic, in the second the initial total momentum is and becomes a null vector.[1] Total isotropic and null vectors are two qualitatively different things. For example, this can be

[1] In this paper, a null-vector has every component equal to zero.

seen from (3.6), which shows that in the one case the energies become zero at $s = 0$, while in the other they become infinite. To exclude (3.27), the selection rule must hold: If the total in-coming momentum is a null-vector, then the total out-going momentum is also a null-vector, and vice versa. This rule is covariant because the concept of a null-vector in momentum space is covariant.

Without using additional assumptions we thus had to introduce two selection rules into the kinematics of inelastic tachyon-tachyon scattering, the one to exclude asymmetric mass change and the other to exclude $m_1 = m_2, m_1' \neq m_2'$ or $m_1 \neq m_2, m_1' = m_2'$, (3.27), and this because of singularities at $s = 0$. We would not have to do this if the alternative approach in Subsection 3.4 applied, since both these cases are characterized by states with unequal masses so that $s' = 0$ would lie outside the physical region.[2] Besides, in this approach constraint (3.4′) is not necessary for limiting the phase space in inelastic tachyon-tachyon scattering.

According to the rule in Subsection 3.4 the rest masses of the final particles are bounded by $|m_1'^2 - m_2'^2| < s$, a restriction that is clearly insufficient for excluding infinite rest masses. But let us assume that each of the m_i''s is allowed to take on some value independently of the given value of the other m_i' (some kind of statistical independence). This assumed ability of each of the m_i''s will conflict with the restriction unless $m_i' < s$. (Another explanation of this bound was given in Lemke, 1977).

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[2] For example, the inelastic reaction $\pi_{ta} + \pi_{ta} \rightarrow \pi_{ta} + \eta_{ta}$ cannot consistently be described within the pure kinematics. (Reactions between leptons do not yield such an example because of L_μ and L_e conservation.) However, if the condition in Subsection 3.4 is satisfied, this reaction can happen. Moreover, this condition suggests regarding the c.m.s. as source rest system (Lemke, 1975, 1977) of the produced η meson.